4

ON THE VARIATION OF THE TRANSPORT FACTOR OF A JUNCTION TRANSISTOR WITH INJECTED CARRIER CONCENTRATION

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ABSTRACT. An attempt has been made to set up a general equation governing the distribution of injected carriers in the base region of a *p-n-p* junction transistor and hence to obtain an expression for the transport factor β. Subject to certain approximations, a relation is derived giving the emitter current density as a function of the concentration of injected carriers and with its help, the transport factor is expressed explicitly in terms of the latter. The expression is critically examined in the light of recombination process both on the surface and in the volume. The results are compared with those suggested by previous workers.

The possible effect of the presence of a significant electronic component of current, across the emitter-base junction on the expression for the transport factor is also considered It is shown that the effect if any would be very small. An electronic component of current however, affects the value of the current amplification factor α and a categorical experimental verdict in favour of one or the other of the different possible modes of recombination is not possible unless the so-called emitter efficiency term can be determined by independent experimental measurement.

1. INTRODUCTION

The current amplification factor of a junction transistor, as given approximately by the product of the so-called transport factor β and the emitter efficiency γ, is known to vary with the emitter current I_e. A large amount of work (Webster 1954, Rittner 1954, Giacoletto 1955, Misawa 1955, Fletcher 1956, Hauri 1956, Matz, 1598 and Kaufmann 1959) has already been done to account for this variation but a satisfactory answer has not yet been obtained. One reason for this is the fact that a rigorous solution of the diffusion equation of a junction transistor holding for all values of I_e is difficult to obtain and only approximate solutions for operation at high injection level have to be inferred by making use of the known results for that at low level and of certain plausible assumptions. The deductions are obviously rather crude and it is not known whether these may be applicable to the intermediate levels of operation An attempt was therefore made to set up a general equation governing the distribution of carriers injected in the base region with a view particularly to obtain an expression for the transport factor β valid over a wide range of operation. In the present paper, an account is given of the results thus obtained for three cases of general interest, viz., (i) when recombination of carriers is confined only to the surface; (ii) when recombination occurs only in the volume and (iii) when recombination occurs both on the surface and in the volume. The results are compared with the values given by the previous workers.

2. GENERAL EQUATION FOR THE DISTRIBUTION OF CARRIERS IN THE BASE REGION

The basic equations governing the one dimensional flow of minority carriers through the base region of a junction transistor are the following :

$$J_n = ne\mu_n E + eD_n \frac{dn}{dx} \qquad \ldots \quad (1)$$

$$J_p = pe\mu_p E + eD_p \frac{dp}{dx} \qquad . \quad (2)$$

where J_n, J_p = electron and hole current densities across the emitter-base boundary.

n, p = electron and hole concentrations in the base region,

μ_n, μ_p = electron and hole mobilities,

D_n, D_p = diffusion constants for electrons and holes

E = electric field in the base region

and e = electronic charge

Confining to transistors of the p-n-p type we note that for high values of emitter efficiency, $J_n \approx 0$. The electric field E can now be eliminated (Webster 1954) between Eqs. (1) and (2) giving

$$J_p = -eD_p \left(1 + \frac{p}{p+N_d} \right) \frac{dp}{dx} \qquad \ldots \quad (3)$$

where N_d = equilibrium donor concentration in the base region

When recombination is present, the time rate of decay of injected hole density in the base region is given by

$$\frac{dp}{dt} = -\frac{p-p_B}{\tau} - \frac{1}{e}\frac{dJ_p}{dx} \qquad \ldots \quad (4)$$

where p_B = thermal equilibrium value of hole density in the base region, and τ = effective lifetime of holes in the base region

Eliminating J_p from Eqs.(3) and (4) one obtains

$$\frac{dp}{dt} = -\frac{p-p_B}{\tau} + D_p \left(1+ \frac{p}{p+N_d} \right) \frac{d^2p}{dx^2} + D_p \frac{N_d}{(p+N_d)^2} \cdot \left(\frac{dp}{dx} \right)^2 \quad \ldots \quad (5)$$

Eq. (4) assumes tacitly that the lifetime τ is independent of the injected minority carrier concentration This assumption is not, however, generally true. Experimental results on the variation of τ with I_e are somewhat conflicting. Results of a recent work (Deb and Daw, 1958) show that in general, leaving out the case involving very low values of I_e, the effective lifetime τ passes through a maximum as the injection level is increased from a low value. An analytical relation suggested to account for this variation is

$$\frac{1}{\tau}=\frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right)+\frac{\nu_B\left(1+\frac{p}{N_d}\right)}{1+\frac{2p}{N_d}}. \qquad \ldots \quad (6)$$

where τ_B = volume recombination lifetime when $p \ll N_d$,

and $1/\nu_B$ = surface recombination lifetime when $p \ll N_d$,

it being assumed that the volume recombination is bimolecular Accepting Eq.(6) one obtains from Eq.(5).

$$\frac{dp}{dt}=-(p-p_B)\left\{\frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right)+\frac{\nu_B\left(1+\frac{p}{N_d}\right)}{1+\frac{2p}{N_d}}\right\}+D_p\left(1+\frac{p}{p+N_d}\right)\frac{d^2p}{dx^2}+$$

$$D_p\left(\frac{dp}{dx}\right)^2\frac{N_d}{(p+N_d)^2} \qquad \ldots \quad (7)$$

Eq. (7) is the general form of diffusion equation valid for all values of I_e, provided the lifetime τ follows the relation given by Eq.(6).

3. SOLUTION OF DIFFUSION EQUATION AND GENERAL EXPRESSION FOR THE TRANSPORT FACTOR

We now proceed to obtain a solution of Eq. (7) deduced in the preceding section. For simplicity, we assume the steady state operating condition. For this, $\frac{dp}{dt}=0$ and

$$\left(1+\frac{p}{p+N_d}\right)\frac{d^2p}{dx^2}+\frac{N_d}{(p+N_d)^2}\left(\frac{dp}{dx}\right)^2-\frac{1}{D_p}\left\{\frac{1}{\tau_B}\left(1+\frac{p}{N_d}\right)+\frac{\nu_B\left(1+\frac{p}{N_d}\right)}{1+\frac{2p}{N_d}}\right\}$$

$$=0 \qquad \ldots \quad (8)$$

This equation is of a standard form and can be integrated easily giving

$$Z\left(\frac{2p+N_d}{p+N_d}\right)^2 = -\frac{2}{D_p}\int\left\{\frac{2p+N_d}{\tau_B N_d}(p-p_B)+\nu_B(p-p_B)\right\}dp + C$$

$$= -\frac{2}{D_p\tau_B}\left[\frac{2p^3}{3N_d}+\left(\frac{1}{2}-\frac{p_B}{N_d}\right)p^2 - p\cdot p_B\right] + \frac{2\nu_B}{D_p}\left[\frac{p^2}{2}-p\cdot p_B\right] + C \quad \ldots \quad (9)$$

where $Z=\left(\frac{dp}{dx}\right)^2$ and C is the constant of integration

Let us now introduce the variable $y=\frac{p}{N_d}$ so that

$$Z=\left(\frac{dp}{dx}\right)^2 = N_d^2\left(\frac{dy}{dx}\right)^2 \quad (10)$$

Substituting Eq. (10) in Eq. (9),

$$\left(\frac{dy}{dx}\right)^2\left(\frac{1+2y}{1+y}\right)^2 = \frac{2}{D_p\tau_B}\left[\frac{2}{3}y^3+\left(\frac{1}{2}-\frac{p_B}{N_d}\right)y^2-\frac{p_B}{N_d}\cdot y\right]$$

$$+\frac{2\nu_B}{D_p}\left[\frac{y^2}{2}-\frac{p_B}{N_d}y\right]+C' \quad . \quad (11)$$

To evaluate C', we utilise the condition that at the collector base junction, the minority carrier concentration is negligible so that

$$\left.\begin{aligned} &y=0 \\ \text{and} \qquad &\frac{dy}{dx}=\left(\frac{dy}{dx}\right)_c \text{ say } = \frac{J_c}{eD_pN_d}\end{aligned}\right\} \quad \ldots \quad (12)$$

where J_c is the collector current density From Eqs (11) and (12), we have

$$C' = \left(-\frac{J_c}{eD_pN_d}\right)^2. \quad .. \quad (13)$$

Again, from Eq. (3), the emitter current density $J_e = J_p$ is given by

$$J_e = -eD_p\left(1+\frac{p_e}{p_e+N_d}\right)\left(\frac{dp}{dx}\right)_e$$

$$= -eD_pN_d\left(\frac{1+2y_e}{1+y_e}\right)\left(\frac{dy}{dx}\right)_e, \quad \ldots \quad (14)$$

where the subscript e refers to the emitter base junction.

From Eqs. (11), (13) and (14), one obtains

$$\left(\frac{J_e}{eD_pN_d}\right)^2 = \frac{2}{D_p\tau_B}\left[\tfrac{2}{3}y_e + \left(\tfrac{1}{2} - \frac{p_B}{N_d}\right)y_e^2 - \frac{p_B}{N_d}y_e\right]$$

$$+\frac{2\nu_B}{D_p}\left[\frac{y_e^2}{2} - \frac{p_B}{N_d}y_e\right] + \left(\frac{J_c}{eD_pN_d}\right)^2, \qquad \dots (15)$$

or
$$J_c = J_e\left[1 - \left(\frac{eD_pN_d}{J_e}\right)^2\frac{2}{D_p\tau_B}\left\{\tfrac{2}{3}y_e^3 + \left(\tfrac{1}{2} - \frac{p_B}{N_d}\right)y_e^2 - \frac{p_B}{N_d}y_e\right\} - \right.$$

$$\left.\left(\frac{eD_pN_d}{J_e}\right)^2\frac{2\nu_B}{D_p}\left\{\frac{y_e^2}{2} - \frac{p_B}{N_d}y_e\right\}\right]^{\frac{1}{2}}. \qquad \dots (16)$$

Differentiating both sides of Eq. (16) with respect to J_e and noting that the transport factor β_{ce} for the grounded base mode is given by $\beta_{ce} = -\dfrac{dJ_c}{dJ_e}$, one obtains

$$\beta_{ce} = \frac{1 - \left[\dfrac{1}{J_e}\dfrac{(eD_pN_d)^2}{D_p\tau_B}\left\{2y_e^2 + \left(1 - 2\dfrac{p_B}{N_d}\right)y_e - \dfrac{p_B}{N_d}\right\} + \dfrac{1}{J_e}\dfrac{(eD_pN_d)^2\nu_B}{D_p}\left(y_e - \dfrac{p_B}{N_d}\right)\right]\dfrac{dy_e}{dJ_e}}{\left[1 - \dfrac{2}{J_e^2}\dfrac{(eD_pN_d)^2}{D_p\tau_B}\left\{\tfrac{2}{3}y_e^3 + \left(\tfrac{1}{2} - \dfrac{p_B}{N_d}\right)y_e^2 - \dfrac{p_B}{N_d}y_e\right\} - \dfrac{2}{J_e^2}\dfrac{(eD_pN_d)^2\nu_B}{D_p}\left\{\dfrac{y_e^2}{2} - \dfrac{p_B}{N_d}y_e\right\}\right]^{\frac{1}{2}}} \qquad (17)$$

which is the general expression for β_{ce}.

4 RELATION BETWEEN THE CURRENT DENSITY AND INJECTED CARRIER CONCENTRATION

The relation for β_{ce} given above [Eq. (17)] has the drawback that it involves both J_e and y_e and a precise relationship between these two parameters is needed to obtain β_{ce} explicitly in terms of either J_e or y_e. Unfortunately, however, a rigorous relation cannot be obtained readily and some approximations have to be made keeping in view, as far as possible, the peculiar conditions which arise under high level condition of operation. Thus noting that under high level

$\left(1 + \frac{y_e}{1 + y_e}\right)$. Webster (1954) obtained, for the case when recombination in the base region is negligible, the relation

$$J_e = \frac{eD_pN_d}{W}\left\{2y_e - ln\,(1 + y_e)\right\} = \frac{eD_pN_dP}{W} \qquad \ldots \quad (18)$$

where $P = 2y_e - ln(1 + y_e)$ and W is the width of the base region of the transistor. In a practical transistor, however, some of the injected carriers are invariably lost by recombination in the base region and Eq (18) needs some modification to take this into account. A simple method of doing this is as follows :

We recall that in the presence of recombination the expression for J_e at low injection level is approximately given by

$$J'_e = \frac{eD_pp_B}{W}\exp.\left(\frac{eV_e}{kT}\right)\frac{\frac{W}{(D_p\tau)^{\frac{1}{2}}}}{\tanh\frac{W}{(D_p\tau)^{\frac{1}{2}}}} \qquad (19)$$

and that in the absence of recombination by

$$J_e' = \frac{eD_pp_B}{W}\exp.\left(\frac{eV_e}{kT}\right), \qquad \ldots \quad (20)$$

where J'_e is the emitter current density at low injection level and V_e the applied emitter to base d.c. potential. It is thus seen that the presence of recombination introduces a factor $\frac{W}{(D_p\tau)^{\frac{1}{2}}} \Big/ \tanh\frac{W}{(D_p\tau)^{\frac{1}{2}}}$ in the expression for J'_e. We assume that to a first approximation this also holds for high injection level. Eq (18) can then be modified as

$$J_e = \frac{eD_pN_dP}{W}\cdot\frac{\frac{W}{(D_p\tau)^{\frac{1}{2}}}}{\tanh\frac{W}{(D_p\tau)^{\frac{1}{2}}}} \qquad \ldots \quad (21)$$

Next we argue that at high injection level the term τ appearing in Eq. (21) is a function of y as given by Eq (6). From Eqs. (6) and (21) we then obtain

$$J_e = \frac{eD_pN_dP}{W}\cdot\frac{W\left(\frac{1+y_e}{D_p\tau_B}\right)^{\frac{1}{2}}}{\tanh W\left(\frac{1+y_e}{D_p\tau_B}\right)^{\frac{1}{2}}} \qquad \ldots \quad (22)$$

when recombination occurs only in the volume and

$$J_e = \frac{eD_pN_dP}{W} \cdot \frac{W\left\{\frac{\nu_B(1+y_e)}{D_p(1+2y_e)}\right\}^{\frac{1}{2}}}{\tanh W\left\{\frac{\nu_B(1+y_e)}{D_p(1+2y_e)}\right\}^{\frac{1}{2}}} \qquad \ldots \quad (23)$$

when recombination is confined only to the surface.

5. RECOMBINATION MECHANISM AND THE VALUE OF THE TRANSPORT FACTOR

An expression giving the value of the transport factor β_{ee} explicitly as a function of y_e can now be derived with the help of Eqs. (17), (22) and (23). It is, however, convenient at this stage to consider separately the effects of surface and volume recombinations. This procedure simplifies mathematical manipulation considerably and also helps to bring out clearly the role of the individual types of recombination processes on the operation of a transistor. Further, as will be shown presently, the results thus obtained are helpful in discussing the case when the two recombination processes are simultaneously operative. We consider first the case of surface recombination.

(i) *Recombination confined only to the surface.* When only surface recombination is operative, $\tau_B = \infty$. Keeping this in mind and substituting in Eq. (17) the value of J_e and $\frac{dy_e}{dJ_e}$ as obtained from Eq. (23), one obtains

$$\beta_{ee} = \frac{1-\left(\tanh^2\frac{W}{L'_{BS}}\right)\frac{y_e}{P}\left[1-\frac{P}{(1+2y_e)^2}\left\{\frac{1}{2}-\frac{\frac{W}{L'_{BS}}}{\sinh\frac{2W}{L'_{BS}}}\right\}\right]^{-1}}{\left[1-\left(\tanh^2\frac{W}{L'_{BS}}\right)\frac{1+2y_e}{1+y_e}\cdot\frac{y_e^2}{P^2}\right]^{\frac{1}{2}}} \qquad \ldots \quad (24)$$

where

$$\frac{1}{L'_{BS}} = \left\{\frac{\nu_B(1+y_e)}{D_p(1+2y_e)}\right\}^{\frac{1}{2}}. \qquad \ldots \quad (25)$$

For usual values of $\frac{W}{L'}$, the term

$$\left[1-\frac{P}{(1+2y_e)^2}\left\{\frac{1}{2}-\frac{\frac{W}{L'_{BS}}}{\sinh\frac{2W}{L'_{BS}}}\right\}\right]^{-1}$$

in the numerator of Eq. (24) is very nearly equal to unity. Eq. (24) can, therefore, be rewritten as

$$\beta_{ce} \approx \frac{1 - \frac{y_e}{P} \tanh^2 \frac{W}{L'_{BS}}}{\left[1 - \frac{y_e^2(1+2y_e)}{P^2(1+y_e)} \tanh^2 \frac{W}{L'_{BS}} \right]^{\frac{1}{2}}} \qquad \dots (26)$$

Eq. (26) predicts a tendency for β_{ce} to increase with y_e, approaching a limiting value of

$$\beta_{ce} \approx 1 - \frac{W^2 \nu_B}{8 D_p} \qquad .. (27)$$

for $y_e >> 1$ and $\frac{W^2 \nu_B}{D_p} << 1$. For $y_e << 1$, Eq. (26) reduces to the form

$$\beta_{ce} \approx 1 - \frac{W^2 \nu_B}{2 D_p}. \qquad \dots (28)$$

If the right hand side of Eq. (26) is expanded binomially, one obtains, neglecting higher order terms,

$$\beta_{ce} \approx 1 - \left(\tanh^2 \frac{W}{L'_S} \right) \left[\frac{y_e}{P} - \frac{y_e^2 \left(\frac{1+2y_e}{1+y_e} \right)}{2P^2} \right]. \qquad \dots (29)$$

Using Eq. (29), the reciprocal of the transport factor β_{cb} for the grounded emitter mode of operation can be obtained as

$$\beta_{cb}^{-1} = \left(\tanh^2 \frac{W}{L'_{BS}} \right) \left[\frac{y_e}{P} - \frac{y_e^2 \left(\frac{1+2y_e}{1+y_e} \right)}{2P^2} \right]$$

$$= \frac{W^2 \nu_B}{2 D_p} \left[\frac{\tanh^2 \frac{W}{L'_{BS}}}{\frac{W^2 \nu_B}{D_p}} \left\{ \frac{2y_e}{P} - \frac{y_e^2 \left(\frac{1+2y_e}{1+y_e} \right)}{P^2} \right\} \right]$$

$$= \frac{W^2 \nu_B}{2 D_p} \cdot k_1(z), \qquad \dots (30)$$

where

$$k_1(z) = \frac{\tanh^2 \dfrac{W}{L'_{BS}}}{\dfrac{W^2 \nu_B}{D_p}} \cdot \left[\frac{2y_e}{P} - \frac{y_e^2 \left(\dfrac{1+2y_e}{1+y_e} \right)}{P^2} \right] \qquad \ldots \quad (31)$$

It is easily shown that for $W << L'$

$$k_1(z) \approx \frac{2y_e(1+y_e)}{P(1+2y_e)} - \left(\frac{y_e}{P} \right)^2 \qquad \ldots \quad (32)$$

$k_1(z)$ may be called the 'fall-off factor' and is analogous to the factors $g(z)$ and $m(z)$ of Webster (1954) and Hauri (1956) respectively. Like these latter factors, $k_1(z)$ tends to unity for $y_e << 1$ but for $y_e >> 1$, the limiting value of this factor is 0.25 as compared to the value of 0.50 for both $g(z)$ and $m(z)$.

In the analysis presented above it has been assumed that L'_B is a function of y_e. Webster has, however, suggested that the surface recombination lifetime is independent of y_e and has ascribed the observed increase in the value of β_{ce} to the increase in the value of the effective diffusion constant. Accepting this point of view, the modified expression for β_{ce} is found to be

$$\beta_{ce} = \frac{1 - \dfrac{y_e}{P} \tanh^2 \dfrac{W}{L_{BS}}}{\left[1 - \dfrac{2\{y_e^2 - y_e + ln(1+y_e)\}}{P^2} \cdot \tanh^2 \dfrac{W}{L_{BS}} \right]^{\frac{1}{2}}}, \qquad \ldots \quad (33)$$

where $\quad \dfrac{1}{L_{BS}} = \left(\dfrac{\nu_B}{D_p} \right)^{\frac{1}{2}}$

For low values of $y_e (y_e << 1)$, Eq. (33) reduces to the form of Eq. (28) and for high values of $y_e (y_e >> 1)$, Eq. (33) simplifies to

$$\beta_{ce} \approx 1 - \frac{W^2 \nu_B}{4D_p} \qquad \ldots \quad (34)$$

which shows that the loss due to surface recombination is halved at high current densities. This is in accordance with the conclusions arrived at by both Webster and Hauri. For intermediate values of y_e, however, the nature of variation of this term can be studied by considering the variation of the 'fall-off factor' which for this case may be written as

$$k_2(z) = \frac{\tanh^2 \frac{W}{L_{BS}}}{\left(\frac{W}{L_{BS}}\right)^2} \left[\frac{2y_e}{P} - \frac{2\{y_e^2 - y_e + ln(1 + y_e)\}}{P^2} \right] \qquad \ldots \quad (35)$$

$$\approx \frac{2y_e}{P} - \frac{2\{y_e^2 - y_e + ln(1 + y_e)\}}{P^2} \qquad \ldots \quad (36)$$

It is easily seen from Eq (36) that the limiting values of the factor $k_2(z)$ for $y_e << 1$ and $y_e >> 1$ are 1 and 0.5 respectively in agreement with those of $g(z)$ and $m(z)$. For intermediate values of y_e, $k_2(z)$ differs slightly from both. Table I compares these values.

TABLE I

Values of the factors $g(z)$, $m(z)$, $k_1(z)$ and $k_2(z)$.

y_e	$g(z)$	$m(z)$	$k_1(z)$	$k_2(z)$
1	2	3	4	5
0.01	0 99	0 99	0.97	1 00
0 05	0 95	0 98	0.89	0.96
0 10	0 92	0.96	0 80	0 94
0.50	0.75	0.86	0.57	0 80
1.00	0.67	0.79	0.44	0 72
2 00	0 60	0 72	0 34	0 64
5.00	0.55	0 64	0.29	0.57
10 00	0 52	0.60	0.26	0 54

(ii) *Recombination confined only to the volume* When the volume recombination term in Eq.(17) plays the dominant role (i.e $\nu_B = 0$), one obtains with the help of Eq. (22) the following expression for β_{oe}

$$\beta_{oe} = \frac{1 - \left(\tanh^2 \frac{W}{L_{BV}'}\right) \frac{y_e}{P} \left[1 - \frac{P}{1+2y_e} \left(\frac{1}{2} - \frac{L_{BV}'}{\sinh \frac{2W}{L_{BV}'}} \right) \right]}{\left[1 - \left(\tanh^2 \frac{W}{L_{BV}'} \right) \frac{y_e^2}{P^2} \left(1 + \frac{4}{3} y_e \right) \right]^{\frac{1}{2}}}, \qquad (37)$$

where
$$\frac{1}{L_{BV}'} = \left(\frac{1+y_e}{D_p\tau_B}\right)^{\frac{1}{2}} . \qquad \dots (38)$$

On making the same assumption as was made in deriving Eq. (26), Eq. (37) can be simplified to

$$\beta_{ce} \approx \frac{1-\frac{y_e}{P}\tanh^2\frac{W}{L_{BV}'}}{\left[1-\left(\frac{y_e}{P}\right)^2\left(1+\frac{4}{3}y_e\right)\tanh^2\frac{W}{L_{BV}'}\right]}, \qquad \dots (39)$$

From Eq (39) it is easily seen that for $y_e << 1$, the expression for β_{ce} reduces to

$$\beta_{ce} \approx 1-\frac{1}{2}\tanh^2\frac{W}{D_p\tau_B} \approx 1-\frac{W^2}{2D_p\tau_B} . \qquad \dots (40)$$

which is the usual expression for low level condition of operation When $y_e >> 1$, the expression for β_{ce} takes the form

$$\beta_{ce} \approx 1-\frac{1}{3}\left(\tanh^2\frac{W}{D_n\tau_n}\right)y_e, \qquad \dots (41)$$

provided $\frac{W^2}{D_n\tau_n}\cdot y_e$

Assuming $\frac{W}{L_{BV}'}, << 1$, one can obtain from Eq. (39) and approximate expression for β_{cb}^{-1} of the form

$$\beta_{cb}^{-1} = \frac{W^2}{D_p\tau_B}\left[\frac{y_e}{P}(1+y_e)\frac{y_e^2}{2P^2}\left(1+\frac{4}{3}y_e\right)\right] \qquad \dots (42)$$

It may be noted here that Matz (1958) has suggested an expression for β_{cb} in the form of an integral which when evaluated gives a relation similar to Eq. (42).

From Eq (42) an expression for the 'fall off factor' for this case can be written down directly. Denoting this by the term $k_3(z)$, we have

$$k_3(z) = \frac{2y_e(1+y_e)}{P} - \frac{y_e^2}{P^2}\left(1+\frac{4}{3}y_e\right) . \qquad \dots (43)$$

It is to be noted that $k_3(z)$, unlike the factors $k_1(z)$ and $k_2(z)$, increases with y_e which shows that the transport factor β_{cb} actually decreases with increasing y_e. Thus the term 'fall off factor' is a misnomer for this case but has been retained for the sake of uniformity. From Eq. (43) it follows easily that $k_3(z)$ is equal to 1 for $y_e << 1$ and is equal to $\frac{2}{3}y_e$ for $y_e >> 1$ Its values in the range $.01 < y_e < 10$ are given in Table II.

TABLE II

Values of the factors $k_3(z)$, $g'(z)$ and $m'(z)$

y_e	$k_3(z)$	$g'(z)$	$m'(z)$
1	2	3	4
0.01	1.01	1.01	1.00
0.05	1.03	1.05	1.01
0.10	1.07	1.10	1.02
0.50	1.35	1.59	1.11
1.00	1.67	2.31	1.21
2.00	2.39	3.90	1.41
5.00	4.47	9.21	2.01
10.00	7.87	18.60	3.00

'Fall off factors' for the cases considered by Webster (1954) and Hauri (1956) may be derived easily by examining the expressions for β_{cb}^{-1} given by these authors and are found to be

$$\left.\begin{aligned} g'(z) &= 1+P && \text{[Webster]} \\ m'(z) &= \{m(z)\}\{1+\tfrac{3}{8}m(z)y_e\} && \text{[Hauri]} \end{aligned}\right\} \qquad \ldots \quad (44)$$

Values of $g'(z)$ and $m'(z)$ as given by Eq (44) for different values of y_e are given in columns 3 and 4 of Table II.

It should be noted however that the type of volume recombination discussed above is the bimolecular one. The other type of interest is monomolecular volume recombination in which the lifetime tends to become independent of y_e. A solution for this case may be obtained in the manner outlined above yielding for β_{ce} the expression

$$\beta_{ce} = \frac{1 - \frac{y_e}{P} \tanh^2 \frac{W}{L_{BV}}}{\left[1 - \frac{2\{y_e^2 - y_e + ln(1+y_e)\}}{P^2} \tanh^2 \frac{W}{L_{BV}}\right]^{\frac{1}{2}}}, \qquad \ldots \quad (45)$$

where $$L_{BV} = (D_p\tau_B)^{\frac{1}{2}}.$$

For the limiting case $y_e >> 1$, Eq. (45) reduces to the form

$$\beta_{ce} \approx 1 - \frac{W^2}{4D_p\tau_B}, \qquad \ldots \quad (46)$$

showing that β_{ce}, after an initial increase with y_e, attains a limiting value. This is in agreement with the conclusions arrived at by both Webster (1954) and Hauri (1956) The increase of β_{ce} with y_e as is observed here is due to the apparent increase of the diffusion constant D_p.

As regards the 'fall off factor' for this case, the formal resemblance between Eq (45) and Eq. (33) evidently suggests that this factor should be identical with the factor $k_2(z)$ A similar argument would show that the 'fall off factors' for the cases treated by Webster and Hauri are identical with the factors $g(z)$ and $m(z)$ respectively.

(iii) *Surface and volume recombination operative simultaneously.* So far the two types of recombination processes have been considered separately for reasons already mentioned. The concept of the "fall off factor", however, enables one to infer the law of variation of β_{ce} with y_e when both the processes are simultaneously operative. We recall (Webster 1954) that at low injection level the expression for β_{cb}^{-1} is given as the sum of three terms corresponding to the contributions of surface recombination, volume recombination and emitter efficiency. Assuming that the emitter efficiency is unity, the expression for β_{cb}^{-1} can be written as

$$\beta_{cb}^{-1} = a\,.\left\{ \begin{matrix} k_1(z) \\ k_2(z) \end{matrix} \right\} + b\,.\left\{ \begin{matrix} k_2(z) \\ k_3(z) \end{matrix} \right\}, \qquad \ldots \quad (47)$$

where a and b are constants depending on the low level values of the surface recombination term and the volume recombination term respectively. The terms within the parentheses in Eq. (47) take into account the possible alternative modes of recombination mentioned earlier.

6. EFFECT OF EMITTER EFFICIENCY ON THE TRANSPORT FACTOR

The analysis given in the preceding section has the drawback that it is based on the assumption $\gamma = 1$, i e $J_n = 0$. In a practical transistor, however, this is not so and the expression for J_p as given by Eq. (3) will consequently be inadequate. When $J_n \neq 0$, one obtains from Eqs. (1) and (2)

$$E = \frac{J_n - ebD_p \dfrac{dp}{dx}}{e\mu_n(p + N_d)}, \qquad \ldots \quad (48)$$

and $$J_p = -eD_p\left(1 + \frac{p}{N_d}\right)\left\{1 - \frac{p}{b}\,\frac{J_n}{J_p(p+N_d)}\right\}^{-1}\frac{dp}{dx} \qquad \ldots \quad (49)$$

where $$b = \frac{\mu_n}{\mu_p}.$$

From Eqs (3) and (49), it is seen that the effective value of the diffusion constant in the latter case is larger than the value obtained with $J_n = 0$. This will make the value of β larger. In actual practice, however, this increase is not appreciable. Thus even if J_n becomes high enough to make $\gamma = 0.9$, the percentage increase in D_p' is not more than 5. The assumption $J_n = 0$, therefore, does not affect appreciably the results reported in Secs. 3—5.

7. DISCUSSION

To summarise the results obtained in the preceding sections we note that the values of the 'fall-off factors' $g(z)$, $m(z)$ and $k_2(z)$ in Table I are in reasonable agreement while those of the factor $k_1(z)$ are much smaller in magnitude for $y_e > 1$. This discrepancy arises because of the fact that in deriving Eq (26), L_{ps}' was assumed to be a function of y_e, its values increasing with increasing values of y_e whereas the other factors, viz. $g(z)$, $m(z)$ and $k_2(z)$, were not based on such assumption A glance at Table I would show that an experimental test of the validity or otherwise of the aforesaid assumption is best made by measuring β at high level of operation This is, however, difficult because of the increased importance of the emitter efficiency term and volume recombination at high level and of the lack of an accurate method of independent measurement of γ (Deb and Daw, 1958). In view of this we can make a check only for the low levels of operation even though at such levels the values of β as predicted by the different theoretical relations in Table I do not differ widely from each other Let us consider the case when y_e is increased from 0 to 0.1. Table I would show that for this change in y_e the percentage increase in β as predicted by the fall off factors $g(z)$, $m(z)$, $k_2(z)$ and $k_1(z)$ are 8, 4, 6 and 20 respectively. Experimentally measured values of change in current amplification factor as obtained for transistor types OC70 and OC602 for the same conditions are 14% for the former and 8.5% for the latter. Bearing in mind the fact that the effects of the volume recombination and emitter efficiency teerms would be to make the experimentally measured percentage change in current amplification factor lower than the true increase in the transport factor β it would appear that the assumption regarding the dependance of surface recombination lifetime on y_e is somewhat justified and that the factor $k_1(z)$ gives the more correct value of fall-off factor than any of the other three

Taking the case of volume recombination we find from Table II that the values of the factor $k_3(z)$ agree quite well with those of $g'(z)$ for $y_e \leqslant 0.1$ At higher values of y_e, however, the increase of $g'(z)$ is faster than that of $k_3(z)$. This is understood if it is recalled that while in the present analysis the apparent increase in the diffusion constant of the minority carriers with the level of injection has been taken account of, Webster in his treatment had ignored it. As regards $m'(z)$, this factor has the slowest variation with y_e and consequently gives the highest value of β_{ce} This is because the assumed nature of bimolecular recombination in this case is different from that given by Eq. (6). Thus in the limiting case $y_e >> 1$, the approximate expressions for β_{ce} as given by Webster, Hauri and Eq. (41) are as follows :

$$\beta_{ce} \approx 1 - \frac{W^2}{2D_p\tau_B}\, y_e \qquad \text{(Webster)} \qquad \ldots \quad (50)$$

$$\beta_{ce} \approx 1 - \frac{W^2}{12D_p\tau_B}\, y_e \qquad \text{(Hauri)} \qquad \ldots \quad (51)$$

$$\beta_{ce} \approx 1 - \frac{W^2}{2D_p\tau_B}\, y_e \qquad \text{(obtained from Eq 41)} \qquad \ldots \quad (52)$$

Here again a categorical experimental verdict in favour of the one or the other of these equations is difficult because of the increasing role of the emitter efficiency term γ under the limiting condition $y_e >> 1$.

It is, however, to be noted that some earlier experimental results (Evans 1956, Deb and Daw 1958) indicate that the effective lifetime passes through a maximum as y_e is increased from a low value. Eq. (6) used in the present analysis is the only relation suggested so far which can account for such a variation. As such Eq (47) undoubtedly deserves careful consideration. Assuming now that Eq. (6) is valid for a practical transistor, one obtains from Eqs. (32), (43) and (47) the following expression for β_{cb}^{-1}

$$\beta_{cb}^{-1} = a\left[\frac{2y_e(1+y_e)}{P(1+2y_e)} - \frac{y_e^2}{P^2}\right] + b\left[\frac{2y_e}{P}(1+y_e) - \frac{y_e^2}{P^2}\left(1+\frac{4}{3}y_e\right)\right] \quad (53)$$

where $$a = \frac{W^2\nu_B}{2D_p} \quad \text{and} \quad b = \frac{W^2}{2D_p\tau}. \qquad \ldots \quad (54)$$

A rough check on the validity of Eq. (53) may be made as follows. The values of τ_B and ν_B for types OC70 and OC602 transistors determined earlier (Deb and Daw, 1958) are given in columns 2 and 3 of Table III. In column 4 are given values of β_{ce} obtained from these values and Eqs. (53) and (54) for $y_e = 1$. Column 5 gives the experimental values of the current amplification factor α and column those of γ using the results of the preceding two columns. In the last column are given the values of $\sigma_e\ L_e$ obtained from the relation

$$\frac{\sigma_b W}{\sigma_e L_e} = \frac{1-\gamma}{\gamma} \qquad \ldots \quad (55)$$

These results are in reasonable agreement with the expected values (Early 1953, Webster 1954 and Kaufmann 1959) and lend support to the validity of Eq. (53)

TABLE III

Values of $\sigma_e L_e$ at $\gamma_e = 1$

Transistor type	τ_B μsec.	$1/\nu_B$ μsec.	β_{ce}	α	γ	$\sigma_e L_e$ mhos
1	2	3	4	5	6	7
OC70	86.4	17 2	0.979	0 973	0 994	0 67
OC602	51.8	26 4	0.979	0.977	0.999	1.60

With regard to the role of the term γ it is found that this does not affect sensibly the utility of Eq (17). It is interesting to note that the presence of an appreciable electronic component of current—apart from decreasing β through a decreased γ value—also increases the same through an increased diffusion constant. In practice, however, the decrease in γ masks completely any increase arising out of this last effect.

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